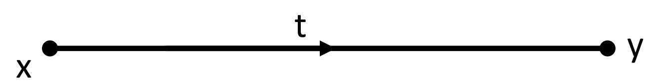
# LECTURE THREE
# SIGNAL FLOW GRAPH

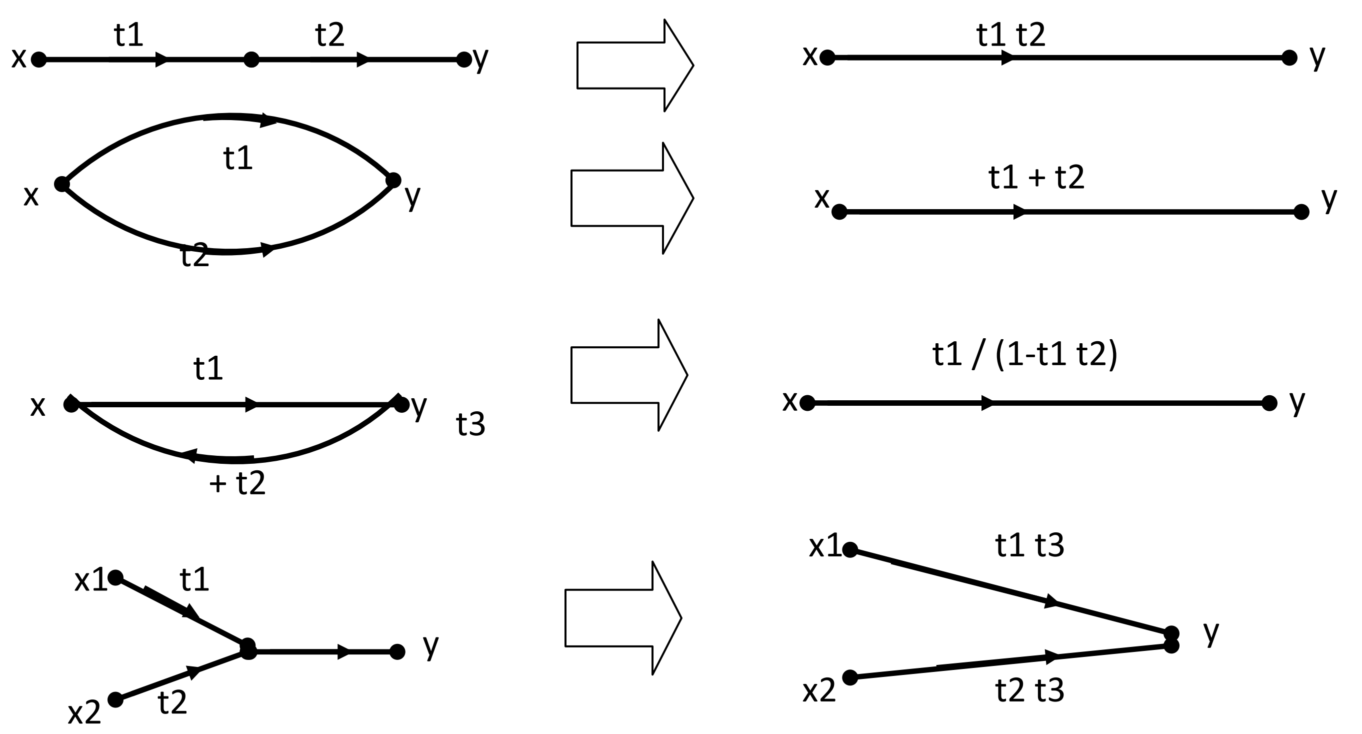
## 3.1 Introduction

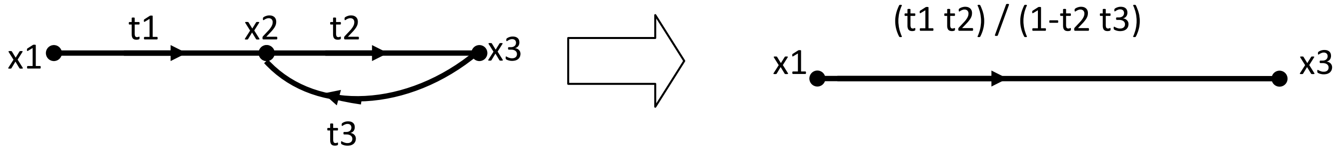
The block diagram reduction technique is tedious and time consuming. Signal flow method gives an alternative approach for finding out transfer function of a control system. Signal flow graph is a network diagram consisting of **<u>nodes</u>**, **<u>branches and arrows</u>**. Nodes represent variables or signals in a system. The nodes are connected by branches and arrows marked on branches indicate direction of flow signal.

If y=t x, then the signal flow graph is;



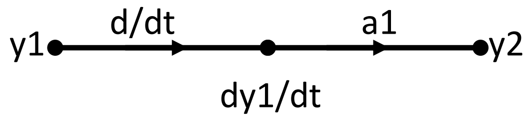

The following rules apply as well for the signal flow graph;



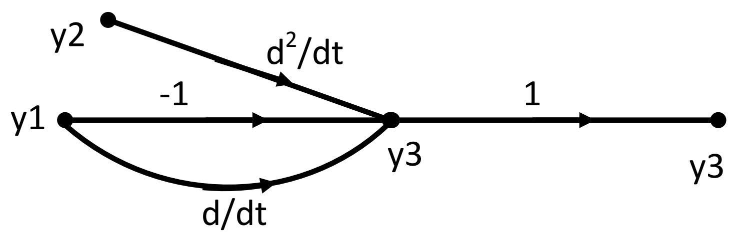



**Examples 3.1:** Draw signal flow graph for the following equations:

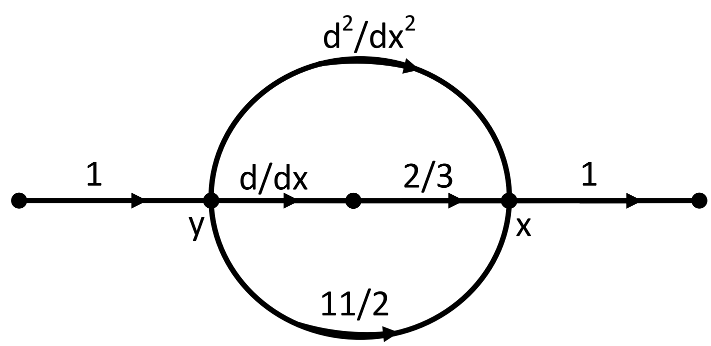
$1 - y_2 = a_1 \frac{dy_1}{dt}$




2- $y_3 = \frac{d^2 y_2}{dt^2} + \frac{dy_1}{dt} - y_1$




3- $\frac{d^2 y}{dx^2} + \frac{2}{3}\frac{dy}{dx} + \frac{11}{2} y = x$

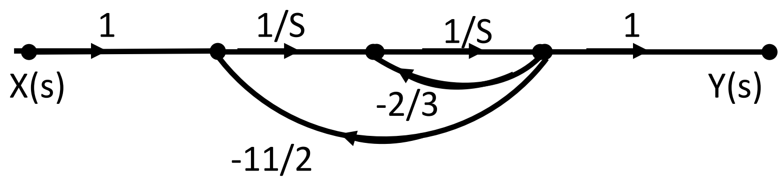



Another solution may be introduced by taking the Laplace transform of the equation as follows;

$$S^2 Y(S) + \frac{2}{3} Y(S) + \frac{11}{2} Y(S) = X(S)$$

$$\frac{Y(S)}{X(S)} = \frac{1}{S^2 + \frac{2}{3}S + \frac{11}{2}} = \frac{1/S^2}{1 + \frac{2}{3S} + \frac{11}{2S^2}} = \frac{1/S^2}{1 - (-\frac{2}{3S} - \frac{11}{2S^2})}$$

Therefore; the signal flow graph is shown;




## 3.2 Mason's Gain Formula

Mason gave a formula relating the output and input. The formula is:

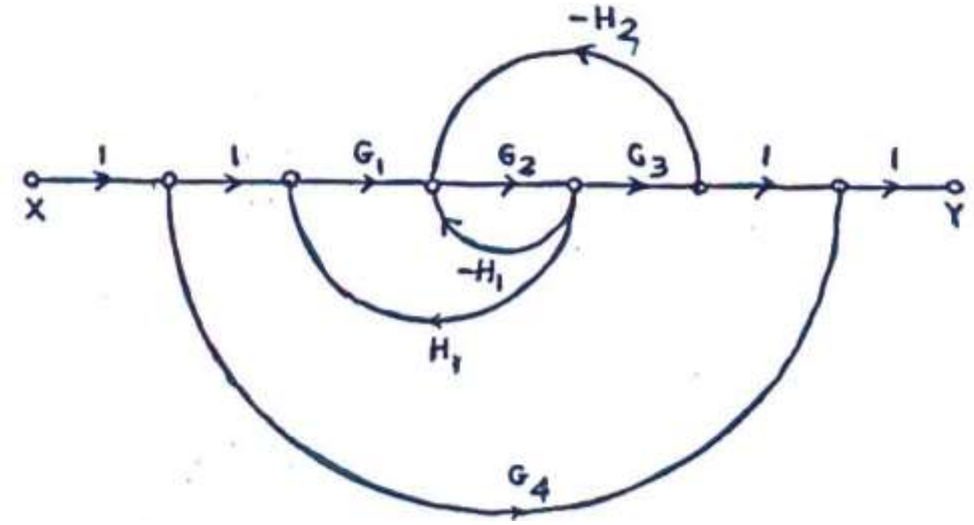
$$\boldsymbol{T = Transfer\ Function = \frac{\sum P_K \Delta_K}{\Delta}}$$

$\Delta$ = 1 – (sum of all individual loops)+(Gain product of all possible combinations of two non-touching loops).

$\Delta_K$= same as for $\Delta$ but formed by loops not touching the $k^{th}$ forward path.

$P_K$ = gain of $k^{th}$ forward paths.

**Example 3.2:** For the system shown, obtain the closed loop transfer function.




**Solution:**

Forward paths:

P1=G1 G2 G3 and P2=G4

Loops:

L1=-G2H1 , L2=G1G2H1, and L3= -G2G3H2

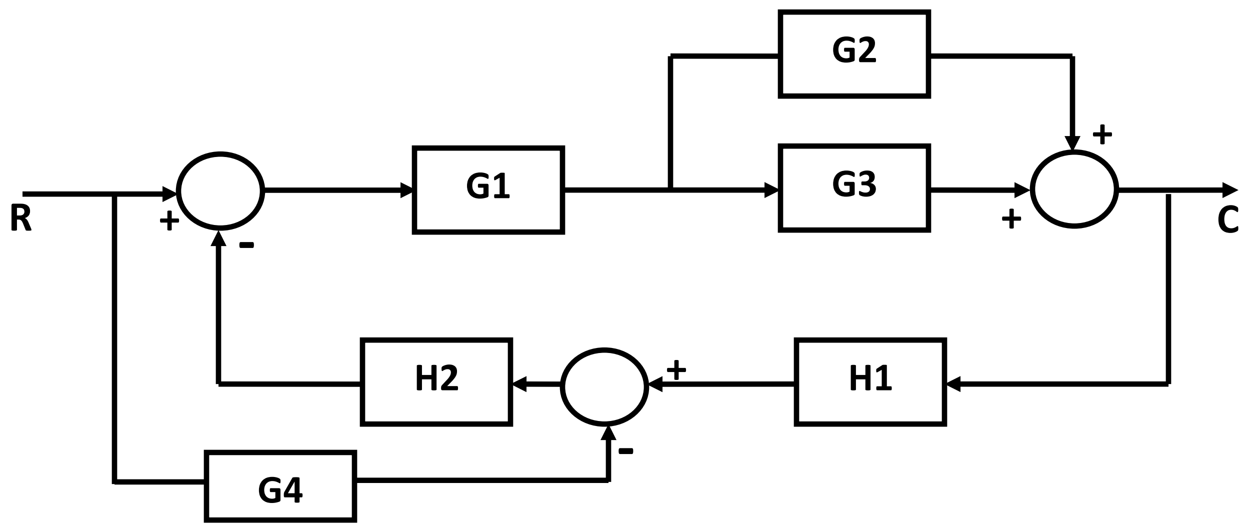
Now; Δ=1-(L1+L2+L3) = 1+ G2H1- G1G2H1+ G2G3H2

Δ1= 1 and Δ2= Δ therefore;

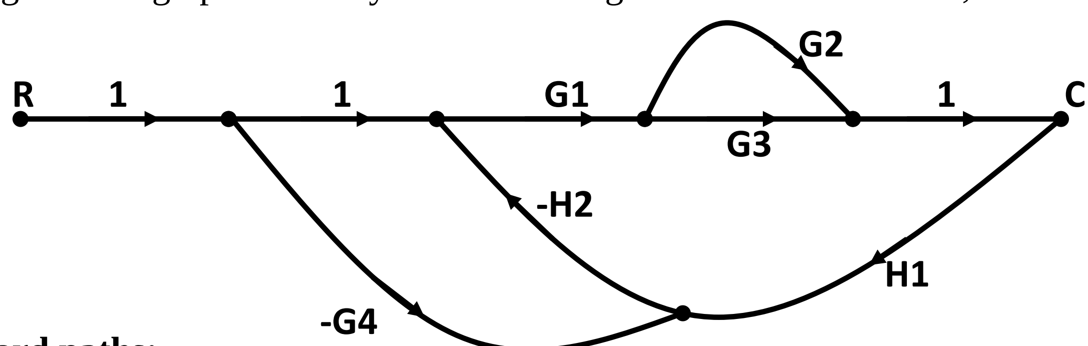
$$\frac{Y}{X} = \frac{P1\ \Delta 1 + P2\ \Delta 2}{\Delta} = \frac{G1\ G2\ G3 + G4(1 + G2\ H1 - G1\ G2\ H1 + G2\ G3\ H2)}{1 + G2\ H1 - G1\ G2\ H1 + G2\ G3\ H2}$$

**Example 3.3:**

Find C/R for the system shown below using signal flow graph technique.




The signal flow graph for the system block diagram above is shown as;




**Forward paths:**

P1=G1 G3 , Δ1=1

P2=G1 G2 , Δ2=1

P3=G1 G3 G4 H2 , Δ3=1

P4 =G1 G2 G4 H2, Δ4=1

**Loops:**

L1= - G1 G3 H1 H2 and L2= -G1 G2 H1 H2

Now;

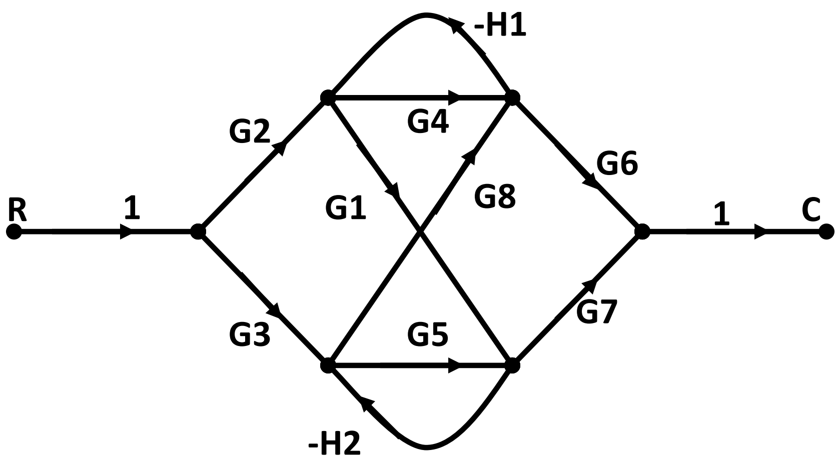
Δ=1- (L1+L2) = 1 + G1 G3 H1 H2 + G1 G2 H1 H2

Therefore;

$$T = \frac{C}{R} = \frac{P1\ \Delta 1 + P2\ \Delta 2 + P3\ \Delta 3 + P4\ \Delta 4}{\Delta}$$

$$T = \frac{G1\ G3 + G1\ G2 + G1\ G3\ G4\ H2 + G1\ G2\ G4\ H2}{1\ +\ \mathrm{G1\ G3\ H1\ H2}\ +\ \mathrm{G1\ G2\ H1\ H2}}$$

**Example 3.4:** Find C/R for the system shown:




**Solution:**

Forward paths:

P1=G2 G4 G6 , P2=G3 G5 G7 , P3= G2 G1 G7 , P4=G3 G8 G6

P5=-G2 G1 H2 G8 G6 , P6=-G3 G8 H1 G1 G7

Loops:

L1=-G4 H1 , L2=-G5 H2 , and L3= G1 H2 G8 H1

Non-touching Loops:

There are one pair of non-touching loops = (G4 H1) (G5 H2)

Now; $\Delta = 1 - (-G4\ H1 - G5\ H2 + G1\ H2\ G8\ H1) + G4\ H1\ G5\ H2$

Δ = 1+ G4 H1 + G5 H2 - G1 H2 G8 H1 + G4 H1 G5 H2

Also;

Δ1=1- (-G5 H2)= 1+ G5 H2 , Δ2=1- (-G4 H1)= 1+ G4 H1 and Δ3= Δ4= Δ5= Δ6=1

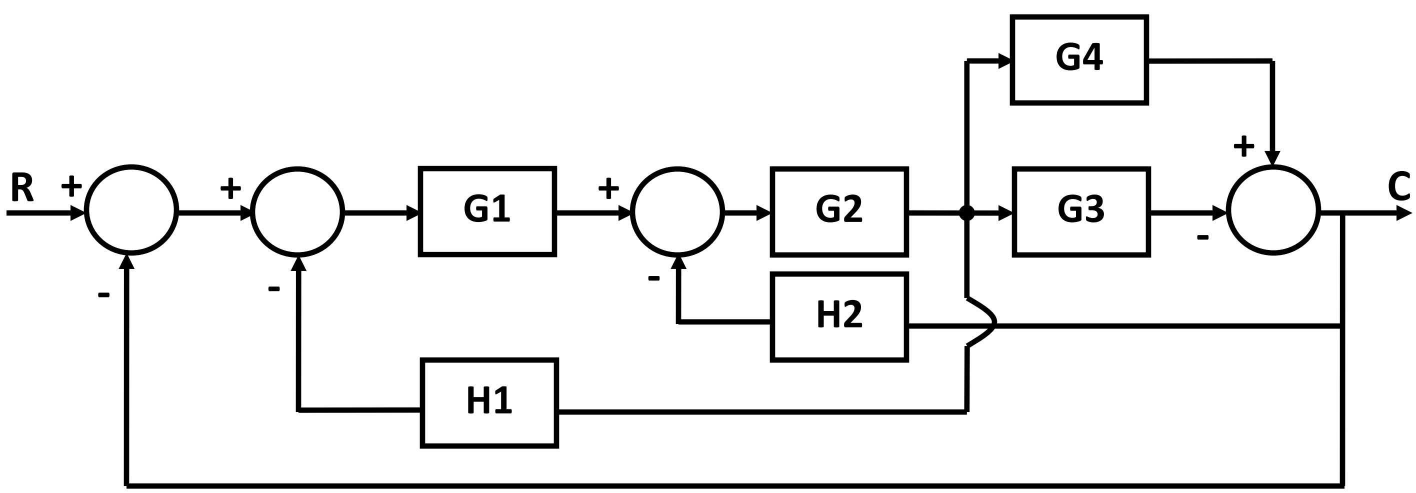
Therefore;

$$T = \frac{C}{R} = \frac{P1\ \Delta 1 + P2\ \Delta 2 + P3\ \Delta 3 + P4\ \Delta 4 + P5\ \Delta 5 + P6\ \Delta 6}{\Delta}$$

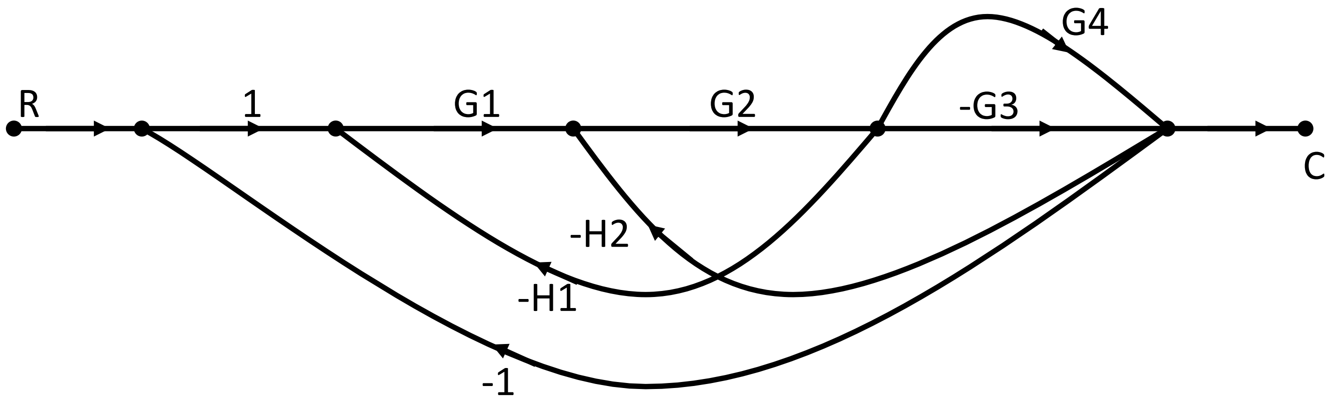
$$T =$$

$$\frac{G2\ G4\ G6(1 + G5\ H2) + G3\ G5\ G7(1 + G4\ H1) + G2\ G1\ G7 + G3\ G8\ G6 - G2\ G6\ G1\ H2 - G3\ G7}{1 + \mathrm{G4\ H1} + \mathrm{G5\ H2} - \mathrm{G1\ H2\ G8\ H1} + \mathrm{G4\ H1\ G5\ H2}}$$

**Example 3.5:** Using Mason's formula method find the transfer function C/R of the below block diagram system.



Converting the block diagram into signal flow graph as shown;




**Forward paths:**

P1=- G1 G2 G3 and P2= G1 G2 G4

**Loops:**

L1= G2 G3 H2 , L2= -G2 G4 H2 , L3= -G1 G2 H1 , L4= G1 G2 G3 and L5= -G1 G2 G4

Now;

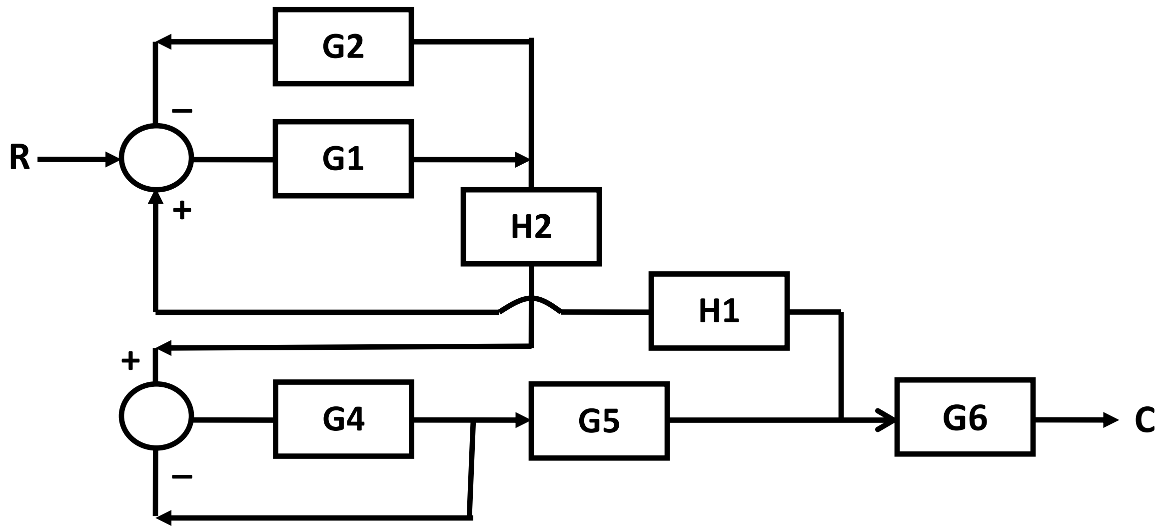
Δ1= 1 and Δ2= 1 and Δ= 1- (L1+L2+L3+L4+L5)

Δ=1 - G2 G3 H2 + G2 G4 H2 + G1 G2 H1 -G1 G2 G3+G1 G2 G4

$$\boldsymbol{T} = \frac{\mathbf{G1\ G2\ G4 - G1\ G2\ G3}}{\mathbf{1\ -\ G2\ G3\ H2\ +\ G2\ G4\ H2\ +\ G1\ G2\ H1\ - G1\ G2\ G3\ + G1\ G2\ G4}}$$

**Example 3.6:**

Using Mason's formula method find the transfer function C/R of the below block diagram system.




Converting the block diagram into signal flow graph as shown;

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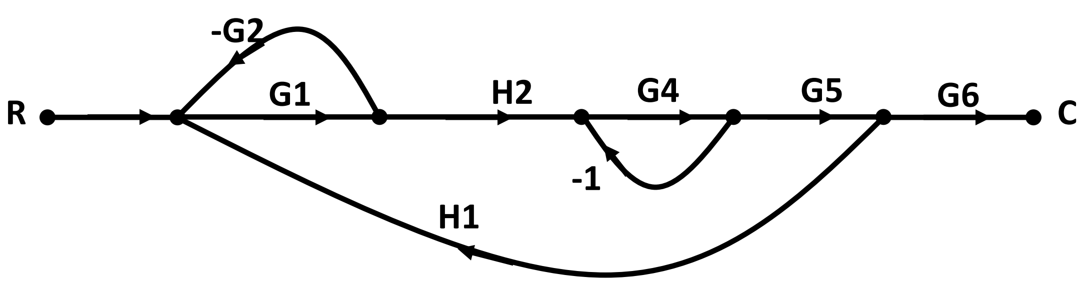



**Forward paths:**

P1= G1 H2 G4 G5 G6 with Δ1=1

**Loops:**

L1= -G1 G2 , L2= -G4, and L3= G1 H2 G4 G5 H1

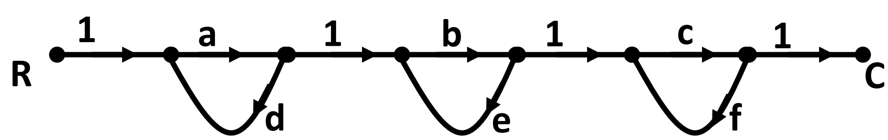
therefore; Δ=1+G1 G2 +G4-G1 H2 G4 G5 H1+ G1 G2 G4

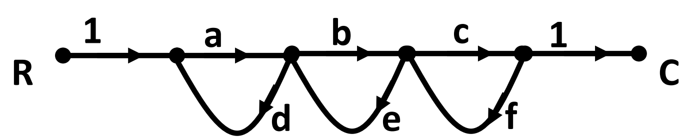
then;

$$\frac{C}{R} = \frac{\text{G1 H2 G4 G5 G6}}{1 + \text{G1 G2} + \text{G4} - \text{G1 H2 G4 G5 H1} + \text{G1 G2 G4}}$$

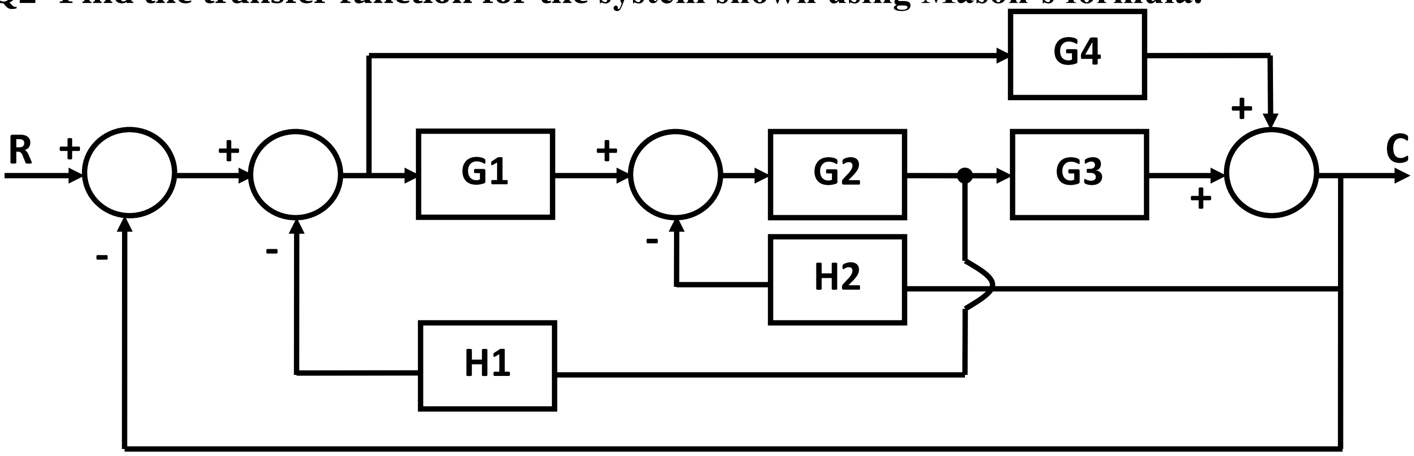
## Exercises:

**Q1- Prove that the two shown control systems have different transfer functions.**

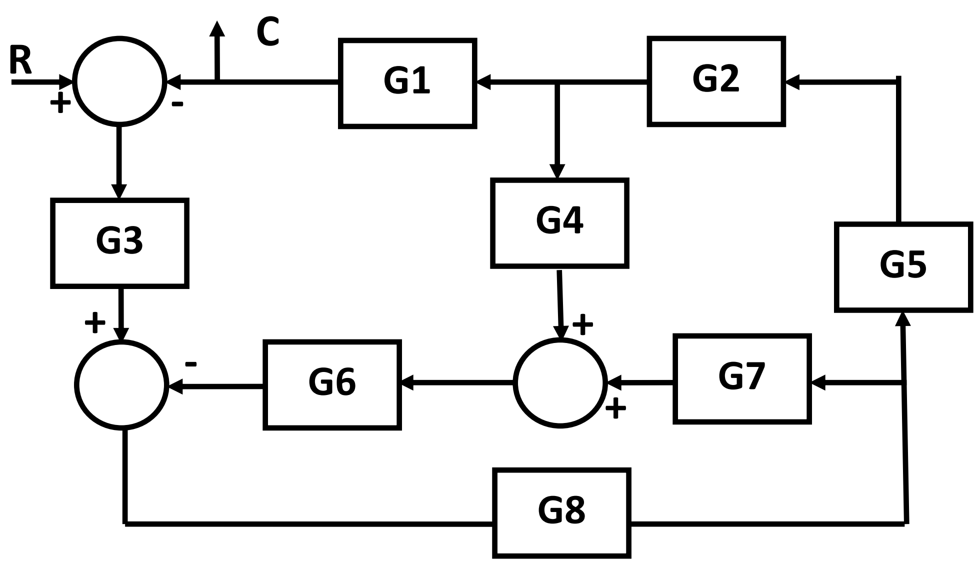


**Q2- Find the transfer function for the system shown using Mason's formula.**




**Q3- Find the transfer function for the system shown using Mason's formula**.




Q4- Find the transfer function for the system shown using Mason's formula.